Project 1: Dynamical Systems

CS 423/523: Complex Adaptive Systems
Assigned Jan 23rd
Due: Feb. 10th, 6:00pm Mountain Time
Version: 1.3

1 Change Log

Ver 1.1 (Jan 24th): fixed two typos.
Ver 1.2 (Jan 30th):

- Made clear that the plots used in Part 1 may not be appropriate for Part 2. However the same questions that were investigated in Part 1 should be investigated in Part 2.
- Figure 2b should be a single plot. It may not contain its own subfigures.
- Fixed a typo (gamma to γ).
- Removed an unnecessary reference to the Jacobian.

Ver 1.3 (Feb 9th):

- Include an author contributions statement.
- Deleted "Discuss the magnitude of attraction to the fixed points" from the Lyapunov section because it was confusing.

2 Introduction

In this project you will work with your assigned partner to explore two dynamical systems. The rules for formatting your paper and a description of the grading rubric is on the course website, be sure to follow them in order to get credit for this project.

A good strategy for writing a scientific paper is think about the figures first. The figures provide a structure around which you can write the rest of the paper and which can guide your methods. Casual readers will usually look at your abstract, conclusions, and figures first before deciding whether to read the rest of the paper. We will follow that strategy in this project description.

Maintain your paper under overleaf. Send us links to your overleaf project via email (bbologa@unm.edu and mfricke@unm.edu).

3 Software

Implement a program that shows the evolution of the systems given below. Your program must generate all the figures or data in the paper. If you post-process your figures to improve their appearance, add text, etc. that is OK. If your program just produces data and you used an external plotting program such as gnuplot to make the figures that is OK too. Just make sure the figures or data produced by your program are recognisably the basis for the figures in your paper.

Make sure you store your program under github and shared the repository with Bianca Bologa (github username: bbologa) and me (github username: gmfricke). Be sure that all the code needed to run your program on the CS lab machines is included in the git repository. If we cannot run your code we cannot give you credit for the assignment.

Include a README.md file in your repository that explains how to run your code and generate the figures used in your paper.

4 Part 1: Dynamical Map

Consider the following two-dimensional dynamical map:

$$x_{t+1} = \rho - x_t^2 + \gamma y_t \tag{1}$$

$$y_{t+1} = x_t. \tag{2}$$

Where ρ and γ are parameters. A trajectory in this system consists of an initial condition (x_0, y_0) and the subsequent iterated values. The dynamics of this system, like the logistic map discussed in class, depend on the choice of parameters ρ and γ and the initial conditions.

5 Dynamical Regimes (Figures 1a, b, c)

Figure 1 of your paper will be three 2D plots of the dynamics with time, t, on the horizontal axis. You will have two vertical axis, one for the x_t values and one for y_t values. Plot your x_t and y_t values in different colors and with different line styles (many reviewers still print papers in B&W).

The three subfigures in Figure 1 should show three different dynamical behaviours: fixed point, limit cycle, and complex. For the complex dynamics examine the system for $t > 5000$.

Include the parameters for each of the subfigures in your Figure 1 caption. Use different parameters from those described in the rest of the paper.

It is *highly unlikely* that any two teams will choose to examine the same values for ρ, γ, x_0, and y_0.

6 Sensitivity to Initial Conditions, Figure 2a,b

Show the sensitivity of the map defined by Eq. (1) and (2) to initial conditions (x_0, y_0). Using the parameter values $\rho = 1.29$ and $\gamma = 0.3$ and initial values selected randomly from the unit interval, $[0, 1]$.

Figure 2 will consist of two subfigures. In Figure 2a show two example time series in which you have varied the initial condition slightly (you may only have to vary x_0 or y_0 to accomplish this).

In Figure 2b make a plot showing how long it takes for trajectories to diverge as a function of the difference in their initial conditions.

7 Bifurcations, Figure 3a,b

Make a bifurcation plot for the map defined by Eq. (1) and (2) in the style of Flake, Figure 10.7 (from our reading).

Vary ρ from 0.5 to 1.4 and set $\gamma = 0.3$. Choose your own initial conditions from the unit interval.

Experiment to find a step size small enough to reveal the structure and a time long enough to show the periodicity produced by each value of ρ that you try. Once you have a plot that shows the detail and periodicity make that Figure 3a. Zoom into an interesting region to show the fine structure in Figure 3b.

If you do not find an interesting pattern consider a different initial value.

8 Lyapunov Exponent, Figure 4

The Lyapunov exponent provides a compact description of the rate of divergence in a chaotic system. Discover the locations of the two attractors in this system.

Plot the Lyapunov exponents for the attractors and identify the values of ρ and γ that produces the largest exponent.

Analyse the stability of the fixed points by calculating the eigenvalues of system.

9 Strange Attractors

Identify any strange attractors in this system.

10 Dissipative vs. Conservative

Discuss whether this system is dissipative or conservative. Describe how you know.

11 Part 2: Dynamical Flow

Try to answer the same questions addressed in Part 1 for the following 3D dynamical flow.

$$\dot{x} = xy - x\gamma + \alpha \tag{3}$$
$$\dot{y} = -(z + x) \tag{4}$$
$$\dot{z} = \beta z + y \tag{5}$$

Use either $\alpha = \beta = 0.2$, and $\gamma = 5.7$ *or* $\alpha = \beta = 0.1$, and $\gamma = 14.$ for your investigation.

This part is intended to be more open ended. Try to find and describe as many interesting dynamical properties of this system as you can. You do not need to produce the same plots as in Part 1. If a particular plot is not appropriate for this system find another way to address the relevant question. There may be other types of plot discussed in class that are more appropriate to use when analysing this system. Add figures as appropriate.

12 Author contributions

Include a contributions statement before the introduction section. The contributions may fall into three categories: analysis, code, and writing. For example your author contribution statement might look like this:

\section*{Author contributions}
J. C. wrote the code that generated Figs. 1, 3, and 5. V. W. wrote the code that generated Fig. 4. Both authors wrote the code that generated Figs. 6 and 7. J.C. wrote sections 1 though subsection 2.3, and section 3.6 of the paper. V. W. wrote subsection 2.4 through 3.5. The authors wrote sections 4 and 5 together. J. C. performed stability analysis for the map and V.W. identified the fixed points for the flow.

13 Notes

Format your paper as described in the project section of the web syllabus. Use the ACM paper template provided. Organise the paper into the following sections:

1. Abstract
2. Introduction
3. Related Work
4. Methods
5. Results
6. Conclusions
7. References

The paper may not exceed 5 single spaced pages including references.